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Less Conservative Antiwindup Synthesis for Time Delay Systems

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***Abstract*—In this paper we address the problem of synthesizing a static antiwindup compensator for linear systems with saturating input, and input and state delays. Based on the use of a Lyapunov-Krasovskii functional, a generalized sector condition and the Finsler Lemma, we propose Linear Matrix Inequalities (LMI) conditions for the synthesis of a static antiwindup compensator that guarantees local (regional) input-to-state stability, as well as asymptotic stability of the closed loop system. The computation of the antiwindup compensator is carried out from the solution of a convex optimization problem: the maximization of the $\mathcal{L}_2$-norm upper bound on the admissible disturbances for which the system trajectories are assured to be bounded. A numerical example illustrates the effectiveness of our methodology, which advances the previous related works towards less conservative results.**

***Index Terms*—Antiwindup, Time Delay, LMI, Continuous Time**

1. Introduction

THE antiwindup compensation is a well-known and efficient technique to cope with undesirable effects on performance and stability which are produced by actuator saturation in control loops. The first results regarding the design of antiwindup compensators were motivated by the degradation of the transient performance induced by saturation in feedback control systems containing integral actions [1], [2]. More recently, the antiwindup problem has been considered in a formal context and a large number of systematic synthesis methods have been proposed [3], [4], [5]. In particular, some of them are based on Linear Matrix Inequalities (LMI) (or *almost* LMI) conditions [6], [7]. The advantage of the LMI-based methods lies in the fact that the antiwindup design can be carried out by solving convex optimization problems. In this case, different optimal synthesis criteria, such as the $\mathcal{L}_2$-gain attenuation or the enlargement of the basin of attraction, can be directly addressed in an optimal way.

Besides the actuator saturation, it is well-known that time delays are present in many control applications and are also source of performance degradation and even instability [8], [9]. However, most of the antiwindup design methods deal with only undelayed systems. The antiwindup compensation for time delay systems, was addressed, for instance, in [10], [11]. In [10], [12] plants subject to input and/or output delays are considered. It should be pointed out that these results apply only to stable open-loop systems and that the approach does not consider systems presenting state delays. In [13], [11], LMI approaches to synthesize a stabilizing static antiwindup compensator for such systems have been proposed. Differently from the classical objective to recover performance, they use the antiwindup compensation to enlarge the region of attraction of the closed loop system. In particular, the action of disturbances and closed loop performance issues are not considered. The dynamic antiwindup synthesis for state-delayed systems has been recently addressed in [14], [15]. The approach in [14] is based on congruence transformations, similar to the ones proposed in [16]. From a Projection Lemma approach, it is shown that the synthesis of a rational compensator can be carried out by LMI conditions [15].

Recently the work [17] has proposed LMI conditions for synthesizing a static antiwindup compensator that assures the closed loop stability for systems with time delay in both input and state, without considering the presence of disturbance of any type on the system. The matrix representation of the existence conditions of an antiwindup compensator is obtained by augmenting its dimension through the insertion of a redundant signal (first order time derivative from states), and then associated with a Finsler multiplier. In fact the applied technique implies forcing almost all of many elements of the Finsler multiplier to be zero, which adds great conservativeness to the results. In [18] a static antiwindup compensator on a delay independent framework was proposed. This means that the delay can be arbitrarily long. Once the LMIs are feasible, the resulting antiwindup compensator assures the closed loop stability. The main drawback of this framework is on the strength of the assurance, leading usually to an under performing result when it exists. Yet later, in [19], a dynamic antiwindup compensator on a delay dependent framework was proposed. The advantages of a dynamic compensator reside on the additional freedom degrees of the compensator to better adjust the performance. The drawback resides on the implementation cost of memory-based antiwindup compensation. In some applications as Active Queue Management (AQM) for Transmission Control Protocol/Internet Protocol (TCP/IP) Routers, the router processor is already busy with packet routing and filtering, and thus it becomes prohibitive.

The present work proposes the synthesis of static antiwindup compensators for input and state delayed systems, with saturating input and subjected to $\mathcal{L}_2$-norm bounded disturbances. The proposed synthesis method exceeds the ones proposed in the literature so far in being less conservative. This is achieved by redundantly representing the closed loop system, equivalently representing its Lyapunov-Krasovskii derivative with the usage of Finsler Lemma, and carrying out the development of LMI representation preserving

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the freedom of its multipliers. Beyond the conservativeness criteria, in a direct comparison with [17], we consider $\mathcal{L}_2$-norm bounded disturbances. Compared to [14], [15] and [18], our framework is delay-dependent, which suits best systems with time delays which are not indefinitely long. Compared to [19], we propose a static antiwindup compensator, which as a matter of implementation is memoryless, thus demanding less resource than the dynamic one proposed in prior studies.

This paper is organized as follows: in Section 2 we present formally the problem under study. In Section 3 we present the lemmas and definitions used all over the development. In Section 4 we ennounce and prove the main theorem of this work. In Section 5 we illustrate the result with a numerical example of interest. Finally, Section 6 casts the concluding remarks of this work.

2. Problem Statement

The following notation is used - For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A^T denotes the transpose of A. $A_{(i)}$ denotes the ith row of matrix A. $\star$ stands for symmetric blocks. I denotes an identity matrix of appropriate order. $\underline{\lambda}(P)$ and $\bar{\lambda}(P)$ denote the minimal and maximal eigenvalues of matrix P, respectively. $\mathcal{C}_\tau = \mathcal{C}([-\tau, 0], \Re^n)$ is the Banach Space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\Re^n$ with the norm $\| \phi \|_c = \sup_{-\tau \le t \le 0} \| \phi(t) \|$. $\| \cdot \|$ refers to the Euclidean vector norm. $\mathcal{C}_\tau^v$ is the set defined by $\mathcal{C}_\tau^v = \{\phi \in \mathcal{C}_\tau; \| \phi \|_c < v,\ v > 0\}$. For $v \in \Re^m$, $sat(v) : \Re^m \to \Re^m$ denotes the classical symmetric saturation function defined as $(sat(v))_{(i)} = sat(v_{(i)}) = sign(v_{(i)}) \min(u_{o(i)}, |v_{(i)}|)$, $\forall i = 1, \ldots, m$, where $u_{o(i)} > 0$ denotes the ith magnitude bound. $blockdiag(\cdots)$ is a block diagonal matrix whose diagonal blocks are the ordered arguments. $He\{A\} = A + A^T$.

Consider the following plant model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\tau) + Bu(t-\tau) + B_\omega \omega(t) \\ y(t) &= C_y x(t) \\ z(t) &= C_z x(t) + D_z u(t) \end{aligned} \tag{1}$$

where vectors $x(t) \in \Re^n$, $u(t) \in \Re^m$, $\omega(t) \in \Re^q$, $y(t) \in \Re^p$, $z(t) \in \Re^l$ are the plant state, input, disturbance, measured output and regulated output, respectively. The time delay τ is assumed to be constant. A, A_d, B, B_ω, C_y, C_z and D_z are matrices of appropriate dimensions.

The plant inputs are supposed to be bounded as follows

$$-u_{o_{(i)}} \le u_{(i)} \le u_{o_{(i)}}, \quad u_{o_{(i)}} > 0, \quad i = 1, \ldots, m \tag{2}$$

The disturbance vector $\omega(t)$ is assumed to be limited in energy, that is, $\omega(t) \in \mathcal{L}_2$. Hence for some scalar δ, $0 \le \frac{1}{\delta} < \infty$, the disturbance $\omega(t)$ is bounded as follows

$$\|\omega(t)\|_2^2 = \int_0^\infty \omega(t)^T \omega(t) dt \le \frac{1}{\delta} \tag{3}$$

In order to control plant (1), we assume that the following controller has been designed for stabilizing the system disregarding the control bounds given in (2)

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + A_{c,d} x_c(t-\tau) + B_c u_c(t) \\ y_c(t) &= C_c x_c(t) + D_c u_c(t) \end{aligned} \tag{4}$$

where $x_c(t) \in \Re^{n_c}$, $u_c(t) \in \Re^p$ and $y_c(t) \in \Re^m$. Matrices A_c, $A_{c,d}$, B_c, C_c and D_c are matrices of appropriate dimensions. The nominal interconnection of controller (4) with plant (1) is given by $u_c(t) = y(t)$ and $u(t-\tau) = y_c(t-\tau)$. Because of the control bounds, the *de facto* control signal to be injected in the system considering the controller output $y_c(t)$ is

$$u(t-\tau) = \text{sat}(y_c(t-\tau))$$

To mitigate the effects of the windup caused by saturation, we add to the state of the previously designed controller an antiwindup signal, thus the controller ends up being described as follows

$$\begin{aligned} \dot{x}_c(t) = &\quad A_c x_c(t) + A_{c,d} x_c(t-\tau) + B_c u_c(t) + E_c \psi(y_c(t)) \\ y_c(t) = &\quad C_c x_c(t) + D_c u_c(t) \end{aligned} \tag{5}$$

where

$$\begin{aligned} \psi(y_c(t)) &= \quad sat(y_c(t)) - y_c(t) \\ \psi(y_c(t-\tau)) &= \quad sat(y_c(t-\tau)) - y_c(t-\tau) \end{aligned}$$

<u>Comment 1</u>: This paper copes with for antiwindup synthesis, and we are not concerned with the computation of nominal controller (4). We assume that it has been previously computed and it would ensure the global asymptotic stability of the closed loop system (1) under the connection $u(t-\tau) = y_c(t-\tau)$.

3. Preliminaries

Through the following matrices

$$\mathbb{A} = \begin{bmatrix} A & 0 \\ B_c C_y & A_c \end{bmatrix}, \mathbb{A}_d = \begin{bmatrix} A_d + BD_c C_y & BC_c \\ 0 & A_{c,d} \end{bmatrix}$$
$$\mathbb{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \mathbb{B}_\omega = \begin{bmatrix} B_\omega \\ 0 \end{bmatrix}, \check{\mathbf{I}} = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix}, \mathbb{D}_z = D_z$$
$$\mathbb{C}_z = \begin{bmatrix} C_z + D_z D_c C_y & D_z C_c \end{bmatrix}, \mathbb{K} = \begin{bmatrix} D_c C_y & C_c \end{bmatrix}$$

the closed loop system can be represented as follows

$$\begin{aligned} \dot{\xi}(t) = &\quad \mathbb{A}\xi(t) + \mathbb{A}_d \xi(t-\tau) + \check{\mathbf{I}} E_c \psi(y_c(t)) \\ &\quad + \mathbb{B}\psi(y_c(t-\tau)) + \mathbb{B}_\omega \omega(t) \\ z(t) = &\quad \mathbb{C}_z \xi(t) + \mathbb{D}_z \psi(y_c(t)) + \mathbb{D}_{z,\omega} \omega(t) \end{aligned} \tag{6}$$

where $\xi(t) = \begin{bmatrix} x(t)^T & x_c(t)^T \end{bmatrix}^T$ and $y_c(t) = \mathbb{K}\xi(t)$

The initial condition of system (6) is denoted by function ϕ_ξ, defined in the interval $[-\tau, 0]$, that is

$$\begin{aligned} \phi_\xi(\theta) &= \begin{bmatrix} x(\theta)^T & x_c(\theta)^T \end{bmatrix}^T \\ &= \begin{bmatrix} \phi_x(\theta)^T & \phi_{x_c}(\theta)^T \end{bmatrix}^T, \forall \theta \in [-\tau, 0], \\ &(t_0, \phi_\xi) \in \Re^+ \times \mathcal{C}_\tau^v \end{aligned}$$

Considering matrices G, $G_\tau \in \Re^{m \times (n+n_c)}$ and the sets

$$\begin{aligned} \mathcal{S}(u_o) = &\quad \{\xi(t) \in \Re^{n+n_c}; \\ &\quad |(\mathbb{K}_{(i)} + G_{(i)})\xi(t)| \le u_{o_{(i)}}, i = 1, \ldots, m\} \\ \mathcal{S}_\tau(u_o) = &\quad \{\xi(t-\tau) \in \Re^{n+n_c}; \\ &\quad |(\mathbb{K}_{(i)} + G_{\tau_{(i)}})\xi(t-\tau)| \le u_{o_{(i)}}, i = 1, \ldots, m\} \end{aligned}$$

we can define now the following lemmas which shall be used along our development.

Lemma 1. *Generalized Sector Condition [20]: If $\xi(t) \in \mathcal{S}(u_o)$ and $\xi(t-\tau) \in \mathcal{S}_\tau(u_o)$ then the relations*

$$\begin{gathered}\psi(y_c(t))^T T\big(\psi(y_c(t)) - G\xi(t)\big) \leq 0 \\ \psi(y_c(t-\tau))^T T_\tau\big(\psi(y_c(t-\tau)) - G_\tau\xi(t-\tau)\big) \leq 0\end{gathered}$$

are verified for any diagonal positive definite matrices T, $T_\tau \in \Re^{m\times m}$.

This Lemma is used as a S-procedure to reduce the conservative of the results. It restricts the solution search from $\Re^{n+n_c}$ to a subset where the condition of this lemma is valid.

Lemma 2. *Jensen Inequality[21]: For any scalar $\tau > 0$, positive definite matrix $Q \in \Re^{m\times m}$ and function $x : [0,\tau] \to \Re^m$ such that the integral is definite, the following inequality holds:*

$$\tau \int_0^\tau x(\theta)^T Q x(\theta) d\theta \geq \left(\int_0^\tau x(\theta)^T d\theta\right) Q \left(\int_0^\tau x(\theta) d\theta\right)$$

This lemma is needed to represent matricially our proposed synthesis conditions. Since we have cross product of closed loop system dynamic terms within an integral, we need to properly separate them, and this is done through this lemma.

Lemma 3. *Finsler Lemma[22]: If there exist a matrix $\mathbf{M}_1 \in \Re^{m\times m}$, a vector $x(t) \in \Re^m$ and a matrix $\mathcal{B} \in \Re^{p\times m}$ such that $x(t)^T\mathbf{M}_1 x(t) < 0, \forall x(t) \neq 0 \,|\, \mathcal{B}x(t) = 0$ is verified, then there exists a matrix $\mathbf{F} \in \Re^{m\times p}$ such that*

$$\mathbf{M}_1 + \mathbf{F}\mathcal{B} + \mathcal{B}^T\mathbf{F}^T < 0$$

In other words, both statements are equivalent.

This is the main tool of our development, and its application comprises its main contribution, which is reducing the cosnervativeness of the synthesis conditions.

4. Main Result

We derive now a result for synthesizing a local stabilizing static antiwindup compensator. Consider the Lyapunov-Krasovskii candidate

$$\begin{aligned} V(t) = \; & \xi(t)^T P\xi(t) + \textstyle\int_{t-\tau}^t \xi(\theta)^T R\xi(\theta) d\theta \\ & + \textstyle\int_{-\tau}^0 \int_{t+\theta}^t \dot{\xi}(\beta)^T Q\dot{\xi}(\beta) d\beta d\theta \end{aligned} \tag{7}$$

where $P = P^T > 0$, $R = R^T > 0$ and $Q = Q^T > 0 \in \Re^{(n+n_c)\times(n+n_c)}$. Then, the following theorem can be stated.

Theorem 1. *If there exist symmetric positive definite $\tilde{P}$, $\tilde{R}$, $\tilde{Q} \in \Re^{(n+n_c)\times(n+n_c)}$, matrices $\tilde{F}_{12}$, $\tilde{F}_{22}$, $\tilde{F}_{32}$, $\tilde{F}_{42} \in \Re^{(n+n_c)\times(n+n_c)}$, $\tilde{F}_{52}$, $\tilde{F}_{62}$, $\tilde{G}$, $\tilde{G}_\tau \in \Re^{m\times(n+n_c)}$, $\tilde{F}_{72} \in \Re^{q\times(n+n_c)}$, scalars γ, α, ζ, and structured matrices $\tilde{F}_{11}$, $\tilde{F}_{21}$, $\tilde{F}_{31}$, $\tilde{F}_{41} \in \Re^{(n+n_c)\times(n+n_c)}$, $\tilde{F}_{51}$, $\tilde{F}_{61} \in \Re^{m\times(n+n_c)}$, where*

$$\begin{aligned} &\tilde{F}_{11} = \begin{bmatrix} \tilde{F}_{11a} & 0 \\ \tilde{F}_{11b} & aI_{n_c}\end{bmatrix}, \tilde{F}_{21} = \begin{bmatrix} \tilde{F}_{21a} & 0 \\ \tilde{F}_{21b} & bI_{n_c}\end{bmatrix} \\ &\tilde{F}_{31} = \begin{bmatrix} \tilde{F}_{31a} & 0 \\ \tilde{F}_{31b} & cI_{n_c}\end{bmatrix}, \tilde{F}_{41} = \begin{bmatrix} \tilde{F}_{41a} & 0 \\ \tilde{F}_{41b} & dI_{n_c}\end{bmatrix} \\ &\tilde{F}_{51} = \begin{bmatrix} \tilde{F}_{51a} & eI_{m\times n_c}\end{bmatrix}, \tilde{F}_{61} = \begin{bmatrix} \tilde{F}_{61a} & fI_{m\times n_c}\end{bmatrix} \\ &\tilde{F}_{71} = \begin{bmatrix} \tilde{F}_{71a} & gI_{q\times n_c}\end{bmatrix}, T = \theta I_m,\; T_\tau = \theta_\tau I_m \end{aligned}$$

and parameters a, b, c, d, e, f, g, θ, θ_τ are determined a priori*, such that the following LMIs are verified*

$$\begin{bmatrix} \Sigma_1 & \Sigma_2 & \Sigma_3 & \Sigma_4 & \Sigma_5 & \Sigma_6 \end{bmatrix} < 0 \tag{8}$$

where

$$\Sigma_1 = \begin{bmatrix} \tau\tilde{Q} - \tilde{F}_{11} - \tilde{F}_{11}^T \\ \tilde{P} - \tilde{F}_{21} + (\mathbb{A}+\mathbb{A}_d)^T\tilde{F}_{11}^T - \tilde{F}_{12}^T \\ -\tilde{F}_{31} + \tilde{F}_{12}^T \\ -\tilde{F}_{41} - \mathbb{A}_d^T\tilde{F}_{11}^T + \tilde{F}_{12}^T \\ -\tilde{F}_{51} + E_c^T\check{\mathbf{I}}^T\tilde{F}_{11}^T \\ -\tilde{F}_{61} + \mathbb{B}^T\tilde{F}_{11}^T \\ -\tilde{F}_{71} + \mathbb{B}_\omega^T\tilde{F}_{11}^T \\ 0 \end{bmatrix}$$

$$\Sigma_2 = \begin{bmatrix} \star \\ \begin{pmatrix} \tilde{R} + \tilde{F}_{21}(\mathbb{A}+\mathbb{A}_d) - \tilde{F}_{22} \\ +(\mathbb{A}+\mathbb{A}_d)^T\tilde{F}_{21}^T - \tilde{F}_{22}^T \end{pmatrix} \\ \tilde{F}_{31}(\mathbb{A}+\mathbb{A}_d) - \tilde{F}_{32} + \tilde{F}_{22}^T \\ \tilde{F}_{41}(\mathbb{A}+\mathbb{A}_d) - \tilde{F}_{42} - \mathbb{A}_d^T\tilde{F}_{21}^T + \tilde{F}_{22}^T \\ T\tilde{G} + \tilde{F}_{51}(\mathbb{A}+\mathbb{A}_d) - \tilde{F}_{52} - E_c^T\check{\mathbf{I}}^T\tilde{F}_{21}^T \\ \tilde{F}_{61}(\mathbb{A}+\mathbb{A}_d) - \tilde{F}_{62} + \mathbb{B}^T\tilde{F}_{21}^T \\ \tilde{F}_{71}(\mathbb{A}+\mathbb{A}_d) - \tilde{F}_{72} + \mathbb{B}_\omega^T\tilde{F}_{21}^T \\ \mathbb{C}_z \end{bmatrix}$$

$$\Sigma_3 = \begin{bmatrix} \star \\ \star \\ -\tilde{R} + \tilde{F}_{32} + \tilde{F}_{32}^T \\ \tilde{F}_{42} - \mathbb{A}_d^T\tilde{F}_{31}^T + \tilde{F}_{32}^T \\ \tilde{F}_{52} + E_c^T\check{\mathbf{I}}^T\tilde{F}_{31}^T \\ T_\tau\tilde{G}_\tau + \tilde{F}_{62} + \mathbb{B}^T\tilde{F}_{31}^T \\ \tilde{F}_{72} + \mathbb{B}_\omega^T\tilde{F}_{31}^T \\ 0 \end{bmatrix}$$

$$\Sigma_4 = \begin{bmatrix} \star \\ \star \\ \star \\ -\frac{1}{\tau}\tilde{Q} - \tilde{F}_{41}\mathbb{A}_d + \tilde{F}_{42} - \mathbb{A}_d^T\tilde{F}_{41}^T + \tilde{F}_{42}^T \\ -\tilde{F}_{51}\mathbb{A}_d + \tilde{F}_{52} + E_c^T\check{\mathbf{I}}^T\tilde{F}_{41}^T \\ -\tilde{F}_{61}\mathbb{A}_d + \tilde{F}_{62} + \mathbb{B}^T\tilde{F}_{41}^T \\ -\tilde{F}_{71}\mathbb{A}_d + \tilde{F}_{72} + \mathbb{B}_\omega^T\tilde{F}_{41}^T \\ 0 \end{bmatrix}$$

$$\Sigma_5 = \begin{bmatrix} \star & \star \\ \star & \star \\ \star & \star \\ \star & \star \\ \begin{pmatrix} -2T\alpha + \tilde{F}_{51}\check{\mathbf{I}}E_c \\ +E_c^T\check{\mathbf{I}}^T\tilde{F}_{51}^T \end{pmatrix} & \star \\ \begin{pmatrix} \tilde{F}_{61}\check{\mathbf{I}}E_c + \\ \mathbb{B}^T\tilde{F}_{51}^T \end{pmatrix} & \begin{pmatrix} -2T_\tau\alpha + \\ \tilde{F}_{61}\mathbb{B} + \\ \mathbb{B}^T\tilde{F}_{61}^T \end{pmatrix} \\ \tilde{F}_{71}\check{\mathbf{I}}E_c + \mathbb{B}_\omega^T\tilde{F}_{51}^T & \begin{pmatrix} \tilde{F}_{71}\mathbb{B} \\ +\mathbb{B}_\omega^T\tilde{F}_{61}^T \end{pmatrix} \\ \mathbb{D}_z & 0 \end{bmatrix}$$

$$\Sigma_6 = \begin{bmatrix} \star & \star \\ \star & \star \\ \star & \star \\ \star & \star \\ \star & \star \\ \star & \star \\ \begin{pmatrix} -I_q + \tilde{F}_{71}\mathbb{B}_\omega \\ +\mathbb{B}_\omega^T \tilde{F}_{71}^T \end{pmatrix} & \star \\ 0 & -\gamma I_l \end{bmatrix}$$

$$\begin{bmatrix} \tilde{P} & \star \\ \alpha\mathbb{K}_{(i)} + \tilde{G}_{(i)} & \zeta u_{o_{(i)}}^2 \end{bmatrix} > 0,\ i = 1,\ldots,m \tag{9}$$

$$\begin{bmatrix} \tilde{P} & \star \\ \alpha\mathbb{K}_{(i)} + \tilde{G}_{\tau_{(i)}} & \zeta u_{o_{(i)}}^2 \end{bmatrix} > 0,\ i = 1,\ldots,m \tag{10}$$

then there exists a static antiwindup compensator $E_c \in \Re^{n_c \times m}$ *as in (5) which ensures that*

1) *The trajectories of the system (6) are bounded for every initial condition in the ball*

$$\mathcal{B}(\beta) = \left\{\phi_\xi \in \mathcal{C}_\tau^v,\ \|\phi_\xi\|_c^2 \le \beta_0\right\}$$

$$\beta_0 = \frac{\beta}{\bar{\lambda}(P) + \tau^2\bar{\lambda}(Q) + \tau\bar{\lambda}(R)}$$

with $\beta = \mu^{-1} - (1/\delta)/\alpha$, $\mu = \zeta/\alpha$, $P = \tilde{P}/\alpha$, $Q = \tilde{Q}/\alpha$ *and* $R = \tilde{R}/\alpha$

2) $\|z(t)\|_2^2 \le \gamma V(0) + \frac{\gamma}{\alpha}\|\omega(t)\|_2^2$

3) *When* $\omega(t) = 0$*, for all initial conditions belonging to*

$$\mathcal{B}(\mu^{-1}) = \left\{\phi_\xi \in \mathcal{C}_\tau^v; \|\phi_\xi\|_c^2 \le \mu_0\right\} \text{and}$$

$$\mu_0 = \frac{\mu^{-1}}{\bar{\lambda}(P) + \tau^2\bar{\lambda}(Q) + \tau\bar{\lambda}(R)}$$

the corresponding trajectories converge asymptotically to the origin.

Proof. Function (7) satisfies $\underline{\lambda}(P)\,\|\xi(t)\|_2^2 \le V(t) \le (\bar{\lambda}(P) + \tau^2\bar{\lambda}(Q) + \tau\bar{\lambda}(R))\,\|\xi_t\|_c^2$, and ξ_t denotes the restriction of $\xi(t)$ to the interval $[t-\tau, t]$ as in [8]. Hence, if the initial condition $\phi_\xi \in \mathcal{B}(\beta)$, it follows that $V(0) \le \beta$. Define now the auxiliary function $\mathcal{J}(t) = \dot{V}(t) - \frac{1}{\alpha}\omega(t)^T\omega(t) + \frac{1}{\gamma}z(t)^T z(t)$. If $\mathcal{J}(t) < 0$, it follows that

$$\begin{aligned}\int_0^T \mathcal{J}(t)dt = \quad & V(T) - V(0) - \tfrac{1}{\alpha}\int_0^T \omega(t)^T\omega(t)dt \\ & + \tfrac{1}{\gamma}\int_0^T z(t)^T z(t)dt < 0\end{aligned} \tag{11}$$

Thus, for any ϕ_ξ belonging to $\mathcal{B}(\beta)$, the above relation implies that $V(T) \le V(0) + \alpha^{-1}\|\omega(t)\|_2^2 \le \beta + (1/\delta)/\alpha \le \mu^{-1}$. Hence, from (7) the satisfaction of (11) implies that $\xi(T)^T P\xi(T) \le V(T) \le \mu^{-1}$, that is, for all $T > 0$ the trajectories of the system do not leave the set $\varepsilon(P, \mu^{-1}) = \left\{\xi \in \Re^{n+n_c},\ \xi(t)^T P\xi(t) \le \mu^{-1}\right\}$ for all $\omega(t)$ satisfying (3) and any initial condition belonging to $\mathcal{B}(\beta)$. Moreover, for $T \to +\infty$, (11) yields $\|z(t)\|_2^2 < \frac{\gamma}{\alpha}\|\omega(t)\|_2^2 + \gamma V(0)$.

Now, from Lemma 1, provided that $\xi(t) \in \mathcal{S}(u_o)$ and $\xi(t-\tau) \in \mathcal{S}_\tau(u_o)$ an upper bound for $\mathcal{J}(t)$ becomes

$$\begin{aligned}&\mathcal{J}(t) \le \dot{\xi}(t)^T P\xi(t) + \xi(t)^T P\dot{\xi}(t) + \xi(t)^T R\xi(t) \\ &-\xi(t-\tau)^T R\xi(t-\tau) + \tau\dot{\xi}(t)^T Q\dot{\xi}(t) + \tfrac{1}{\gamma}z(t)^T z(t) \\ &-\int_{t-\tau}^t \dot{\xi}(\theta)^T Q\dot{\xi}(\theta)d\theta - 2\psi(y_c(t-\tau))^T T_\tau\psi(y_c(t-\tau)) \\ &+\psi(y_c(t))^T TG\xi(t) + \xi(t)^T G^T T\psi(y_c(t)) \\ &-2\psi(y_c(t))^T T\psi(y_c(t)) + \psi(y_c(t-\tau))^T T_\tau G_\tau\xi(t-\tau) \\ &+\xi(t-\tau)^T G_\tau^T T_\tau\psi(y_c(t-\tau)) - \tfrac{1}{\alpha}\omega(t)^T\omega(t)\end{aligned}$$

Applying Lemma 2 on the integral term on the right side of the above inequality and defining $\int_{t-\tau}^t \dot{\xi}(\theta)d\theta = \xi(t) - \xi(t-\tau)$, we have

$$\begin{aligned}&-\int_{t-\tau}^t \dot{\xi}(\theta)^T Q\dot{\xi}(\theta)d\theta \le \cdots \\ &-\left(\int_{t-\tau}^t \dot{\xi}(\theta)^T d\theta\right)^T \tfrac{1}{\tau}Q\left(\int_{t-\tau}^t \dot{\xi}(\theta)d\theta\right)\end{aligned}$$

Thus, we have

$$\begin{aligned}&\left(\int_{t-\tau}^t \dot{\xi}(\theta)^T d\theta\right)^T \tfrac{1}{\tau}Q\left(\int_{t-\tau}^t \dot{\xi}(\theta)d\theta\right) = \cdots \\ &\big(\xi(t) - \xi(t-\tau)\big)^T \tfrac{1}{\tau}Q\big(\xi(t) - \xi(t-\tau)\big)\end{aligned}$$

and $\mathcal{J}(t)$ becomes bounded by

$$\begin{aligned}&\mathcal{J}(t) \le \dot{\xi}(t)^T P\xi(t) + \xi(t)^T P\dot{\xi}(t) + \xi(t)^T R\xi(t) \\ &-\xi(t-\tau)^T R\xi(t-\tau) + \tau\dot{\xi}(t)^T Q\dot{\xi}(t) - \tfrac{1}{\alpha}\omega(t)^T\omega(t) \\ &-(\xi(t) - \xi(t-\tau))^T \tfrac{1}{\tau}Q(\xi(t) - \xi(t-\tau)) + \tfrac{1}{\gamma}z(t)^T z(t) \\ &+\psi(y_c(t))^T TG\xi(t) - 2\psi(y_c(t-\tau))^T T_\tau\psi(y_c(t-\tau)) \\ &-2\psi(y_c(t))^T T\psi(y_c(t)) + \psi(y_c(t-\tau))^T T_\tau G_\tau\xi(t-\tau) \\ &+\xi(t-\tau)^T G_\tau^T T_\tau\psi(y_c(t-\tau)) + \xi(t)^T G^T T\psi(y_c(t))\end{aligned}$$

with a vector $\eta(t)$

$$\begin{bmatrix} \dot{\xi}(t)^T & \xi(t)^T & \xi(t-\tau)^T & (\xi(t) - \xi(t-\tau))^T & \cdots \\ \cdots & \psi(y_c(t))^T & \psi(y_c(t-\tau))^T & \omega(t)^T & \end{bmatrix}^T$$

Since we want to ensure $\mathcal{J}(t) < 0$, it suffices ensuring its upper bound as negative definite. Once we can matricially represent it as $\eta(t)^T\mathbf{M}_1\eta(t) < 0$, we now look forward to ensure $\mathbf{M}_1 < 0$. $\mathbf{M}_1$ is given as follows:

$$\begin{bmatrix} \frac{\tau}{2}Q & 0 & 0 & 0 & 0 & 0 & 0 \\ P & \frac{1}{2}R & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2}R & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{2\tau}Q & 0 & 0 & 0 \\ 0 & TG & 0 & 0 & -T & 0 & 0 \\ 0 & 0 & T_\tau G_\tau & 0 & 0 & -T_\tau & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2\alpha}I_q \end{bmatrix}$$

$$+(\star) + \tfrac{1}{\gamma}\mathcal{C}_z^T\mathcal{C}_z$$

where $\mathcal{C}_z = \begin{bmatrix} 0 & \mathbb{C}_z & 0 & 0 & \mathbb{D}_z & 0 & 0 \end{bmatrix}$

We now apply Lemma 3, which shall reduce greatly the conservativeness of our condition $\mathbf{M}_1 < 0$, for as we do not want to ensure $\mathbf{M}_1 < 0$ for any $\eta(t)$, but only for the *de facto* trajectories of the closed loop system (6). Thus we define matrix $\mathcal{B}$ as

$$\mathcal{B} \triangleq \begin{bmatrix} -I & \mathbb{A} + \mathbb{A}_d & 0 & -\mathbb{A}_d & \check{\mathbf{I}}E_c & \mathbb{B} & \mathbb{B}_\omega \\ 0 & -I & I & I & 0 & 0 & 0 \end{bmatrix}$$

and from Lemma 3 we now look forward to ensure $\mathbf{M}_1 + \mathbf{F}\mathcal{B} + \mathcal{B}^T\mathbf{F}^T < 0$. We choose $\mathbf{F}$ to be

$$\begin{bmatrix} F_{i1} & F_{i2} \\ \vdots & \vdots \end{bmatrix}, \; i = 1, \ldots, 7$$

and we can write $\mathbf{F}\mathcal{B}$ as

$$\begin{bmatrix} -F_{11} & F_{11}(\mathbb{A}+\mathbb{A}_d)-F_{12} & F_{12} & -F_{11}\mathbb{A}_d+F_{12} \\ -F_{21} & F_{21}(\mathbb{A}+\mathbb{A}_d)-F_{22} & F_{22} & -F_{21}\mathbb{A}_d+F_{22} \\ -F_{31} & F_{31}(\mathbb{A}+\mathbb{A}_d)-F_{32} & F_{32} & -F_{31}\mathbb{A}_d+F_{32} \\ -F_{41} & F_{41}(\mathbb{A}+\mathbb{A}_d)-F_{42} & F_{42} & -F_{41}\mathbb{A}_d+F_{42} \\ -F_{51} & F_{51}(\mathbb{A}+\mathbb{A}_d)-F_{52} & F_{52} & -F_{51}\mathbb{A}_d+F_{52} \\ -F_{61} & F_{61}(\mathbb{A}+\mathbb{A}_d)-F_{62} & F_{62} & -F_{61}\mathbb{A}_d+F_{62} \\ -F_{71} & F_{71}(\mathbb{A}+\mathbb{A}_d)-F_{72} & F_{72} & -F_{71}\mathbb{A}_d+F_{72} \end{matrix} \cdots$$

$$\cdots \begin{matrix} F_{11}\check{\mathbf{I}}E_c & F_{11}\mathbb{B} & F_{11}\mathbb{B}_w \\ F_{21}\check{\mathbf{I}}E_c & F_{21}\mathbb{B} & F_{21}\mathbb{B}_w \\ F_{31}\check{\mathbf{I}}E_c & F_{31}\mathbb{B} & F_{31}\mathbb{B}_w \\ F_{41}\check{\mathbf{I}}E_c & F_{41}\mathbb{B} & F_{41}\mathbb{B}_w \\ F_{51}\check{\mathbf{I}}E_c & F_{51}\mathbb{B} & F_{51}\mathbb{B}_w \\ F_{61}\check{\mathbf{I}}E_c & F_{61}\mathbb{B} & F_{61}\mathbb{B}_w \\ F_{71}\check{\mathbf{I}}E_c & F_{71}\mathbb{B} & F_{71}\mathbb{B}_w \end{bmatrix}$$

Let $\mathbf{M}_2 = \mathbf{M}_1 + \mathbf{F}\mathcal{B} + \mathcal{B}^T\mathbf{F}^T$, where $\mathbf{M}_2$ is as follows

Now, to assure $\mathcal{J}(t) < 0$ it is sufficient to satisfy $\mathbf{M}_2 < 0$. Hence we multiply pre and post $\mathbf{M}_2$ by $blockdiag\{\sqrt{\alpha}, \cdots, \sqrt{\alpha}\}$, thereby making the following variable changes. By applying the Schur complement, we reach (8), the first LMI condition of this theorem.

$$\begin{aligned} &\tilde{P} = \alpha P,\ \tilde{R} = \alpha R,\ \tilde{Q} = \alpha Q,\ \tilde{G} = \alpha G,\ \tilde{G}_\tau = \alpha G_\tau \\ &\tilde{F}_{ij} = \alpha F_{ij},\ i = 1, \ldots, 7, \quad j = 1, 2 \end{aligned} \tag{12}$$

As stated in the beginning of this proof, if $\mathcal{J}(t) < 0$, from (11) we can conclude that the trajectories of $\xi(t)$ never leave the ellipsoid $\varepsilon(P, \mu^{-1})$ for all $t > 0$, provided that $\xi(t) \in \mathcal{S}(u_o)$ and $\xi(t-\tau) \in \mathcal{S}_\tau(u_o)$. Hence, the inclusion of $\varepsilon(P, \mu^{-1}) \subset \mathcal{S}(u_o) \cap \mathcal{S}_\tau(u_o)$ assures that the conditions of Lemma 1 hold. Thus we add to our LMI set the following

$$\begin{bmatrix} P & \star \\ \mathbb{K}_{(i)} + G_{(i)} & \mu u^2_{o_{(i)}} \end{bmatrix} > 0, i = 1, \ldots, m$$
$$\begin{bmatrix} P & \star \\ \mathbb{K}_{(i)} + G_{\tau_{(i)}} & \mu u^2_{o_{(i)}} \end{bmatrix} > 0, i = 1, \ldots, m$$

Pre and post multiplying the above matrices by $blockdiag\{\sqrt{\alpha}, \cdots, \sqrt{\alpha}\}$, considering $\zeta = \mu\alpha$ and the variable changes pointed out in (12) we obtain LMIs (9) and (10).

As the verification of (9) and (10) assures the validity of Lemma 1, the simultaneous verification of (8)-(10) assures that $\mathcal{J}(t) < 0$, $\forall \omega(t)$ such that $\|\omega(t)\|_2^2 \leq \frac{1}{\delta}$ and $\forall \phi_\xi \in \mathcal{B}(\beta)$. This concludes the proof of Theorem 1. □

5. Numerical Example

Consider the TCP/IP router queue model with a proportional-integral (PI) controller from [23], [24]. The state variables represent the congestion window and the queue size, respectively, the disturbance accounts for User Datagram Protocol (UDP) traffic, and the input is the packet discarding probability. The setup is $N = 60$, $\tau = 0.246$, $C = 3750$, $p_0 = 0.008$ and $q_0 = 175$. Below is given the corresponding plant model.

$$\begin{aligned} &A = \begin{bmatrix} -0.2644 & -0.0044 \\ 243.9024 & -4.065 \end{bmatrix}; \quad B = \begin{bmatrix} -480.47 \\ 0 \end{bmatrix} \\ &A_d = \begin{bmatrix} -0.2644 & -0.0044 \\ 0 & 0 \end{bmatrix}; \quad B_\omega = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &C_y = C_z = \begin{bmatrix} 0 & 1 \end{bmatrix}; \quad D_{z,\omega} = 0; \quad D_z = 0 \\ &A_c = 0; \quad A_{c,d} = 0; \quad B_c = 1; \quad C_c = 9.7811 \times 10^{-6} \\ &D_c = 18.4972 \times 10^{-6}; \quad u_o = 0.991; \quad \tau = 0.246 \end{aligned}$$

The following parameters are found by grid search. The obtained parameters are: $a = 1.20$, $b = 0.50$, $c = 0.50$, $d = 1.55$, $e = -1.00$, $f = -1.00$, $g = -0.20$, $\theta = 1.00$ and $\theta_\tau = 1.00$. Resulting $\frac{1}{\delta} = 3.7765 \times 10^{11}$ and the corresponding antiwindup compensator is $E_c = 6.8809 \times 10^4$.

In the simulation, we apply a step function with amplitude of 6×10^5, being applied to the plant in the interval $[5, 35]$. The response of the system is depicted in Figure 1. Where we compare the system response among PI controller, static antiwindup and dynamic antiwindup compensator [19]. We can clearly see that the queue size tracks the equilibrium point much faster when our compensator is used.

It should be noted that our goal is to return to the equilibrium point as fast as possible. We do not acknoledge other metrics to be as effective as this one in delay control of data networks.

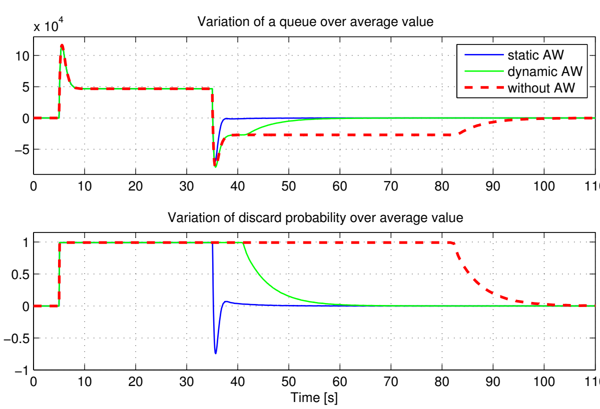

Fig. 1. TCP/IP Router Queue Size X Discarding Probability

6. Conclusion

In this work we have presented a methodology for synthesizing static antiwindup compensators for systems subject to time delays and input saturation. Conditions in an LMI form have been proposed in order to compute an antiwindup compensator, ensuring that the trajectories are bounded for $\mathcal{L}_2$-norm bounded disturbances, while ensuring the internal asymptotic stability of the closed loop system. A numerical example illustrates our results. Clearly our methodology outpaces the previous state of the art in this matter.

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$$
\mathbf{M}_2 = \begin{bmatrix}
\frac{\tau}{2}Q - F_{11} & \begin{pmatrix} -F_{12}+ \\ F_{11}(\mathbb{A}+\mathbb{A}_d) \end{pmatrix} & F_{12} & -F_{11}\mathbb{A}_d + F_{12} & F_{11}\check{\mathbf{I}}E_c & F_{11}\mathbb{B} & F_{11}\mathbb{B}_w \\
P - F_{21} & \begin{pmatrix} \frac{1}{2}R - F_{22}+ \\ F_{21}(\mathbb{A}+\mathbb{A}_d) \end{pmatrix} & F_{22} & -F_{21}\mathbb{A}_d + F_{22} & F_{21}\check{\mathbf{I}}E_c & F_{21}\mathbb{B} & F_{21}\mathbb{B}_w \\
-F_{31} & F_{31}(\mathbb{A}+\mathbb{A}_d) - F_{32} & -\frac{1}{2}R + F_{32} & -F_{31}\mathbb{A}_d + F_{32} & F_{31}\check{\mathbf{I}}E_c & F_{31}\mathbb{B} & F_{31}\mathbb{B}_w \\
-F_{41} & F_{41}(\mathbb{A}+\mathbb{A}_d) - F_{42} & F_{42} & \begin{pmatrix} -\frac{1}{2\tau}Q + F_{42} \\ -F_{41}\mathbb{A}_d \end{pmatrix} & F_{41}\check{\mathbf{I}}E_c & F_{41}\mathbb{B} & F_{41}\mathbb{B}_w \\
-F_{51} & \begin{pmatrix} TG - F_{52}+ \\ F_{51}(\mathbb{A}+\mathbb{A}_d) \end{pmatrix} & F_{52} & -F_{51}\mathbb{A}_d + F_{52} & -T + F_{51}\check{\mathbf{I}}E_c & F_{51}\mathbb{B} & F_{51}\mathbb{B}_w \\
-F_{61} & F_{61}(\mathbb{A}+\mathbb{A}_d) - F_{62} & T_\tau G_\tau + F_{62} & -F_{61}\mathbb{A}_d + F_{62} & F_{61}\check{\mathbf{I}}E_c & -T_\tau + F_{61}\mathbb{B} & F_{61}\mathbb{B}_w \\
-F_{71} & F_{71}(\mathbb{A}+\mathbb{A}_d) - F_{72} & F_{72} & -F_{71}\mathbb{A}_d + F_{72} & F_{71}\check{\mathbf{I}}E_c & F_{71}\mathbb{B} & -\frac{1}{2\alpha} + F_{71}\mathbb{B}_w
\end{bmatrix}
$$

$$
+(\star) + \frac{1}{\gamma}\mathcal{C}_z^T\mathcal{C}_z
$$

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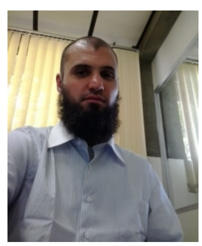

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